



## Measurement of Inner Bremsstrahlung in Polarized Muon Decay with MEG

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### Abstract

A muon decay accompanied by a photon through the inner Bremsstrahlung process ( $\mu \rightarrow e\nu\bar{\nu}\gamma$ , radiative muon decay) produces a time-correlated pair of positron and photon which becomes one of the main backgrounds in the search for  $\mu \rightarrow e\gamma$  decay. This channel is also an important probe of timing calibration and cross-check of whole the experiment. We identified a large sample ( $\sim 13000$ ) of radiative muon decays in MEG data sample. The measured branching ratio in a region of interest in the  $\mu \rightarrow e\gamma$  search is consistent with the standard model prediction. It is also the first measurement of the decay from polarized muons. The precision measurement of this mode enables us to use it as one of the normalization channels of  $\mu \rightarrow e\gamma$  decay successfully reducing its uncertainty to less than 5%.

**Keywords:** muon decay, MEG

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### 1. Introduction

In the standard model of particle physics (SM), muons decay through the weak interaction;  $\mu \rightarrow e\nu\bar{\nu}$  (Michel decay). A photon can accompany this decay through the inner Bremsstrahlung process, and this de-

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cay is called the radiative muon decay;  $\mu \rightarrow e\nu\bar{\nu}\gamma$  (RMD). RMD produces one of the main sources of high energy photon becoming the crucial background in the search for the lepton flavor violating decay  $\mu \rightarrow e\gamma$  when it is observed in coincidence with a positron accidentally overlapping. RMD can also be a background by itself when the neutrinos carry away little energy.

The MEG experiment has been searching for the  $\mu^+ \rightarrow e^+\gamma$  decay since 2008. A new upper limit on the branching ratio based on a sample in 2009–2011 is given [1]. A detailed description of the experiment can be found in [2, 3]. MEG uses surface muons which are fully polarized at the origin. The depolarization mechanisms along the beam-line and in the stopping target are relatively small and under control. The polarization of decaying muons are measured to be  $P = 0.89 \pm 0.04$  from the distribution of positrons from the decays.

Studying RMD is important not only because it is a source of background in  $\mu^+ \rightarrow e^+\gamma$  search but because of the following: it allows calibration for the positron-photon relative timing and measurement of the resolution; and it gives a powerful tool of internal check of the  $\mu^+ \rightarrow e^+\gamma$  analysis. In addition, it could give a test of the weak interaction. Although measurements of RMD were obtained by other experiments [4], MEG data in particular give us a unique opportunity to measure RMD at its kinematic edge owing to the largest amount of the muon decay sample, and to study RMD from polarized muon decay which has never been measured.

## 2. Distribution of RMD

The RMD differential branching ratio was calculated by several authors. Its form is given in [5] for the general structure of interaction. Within the  $V - A$  interaction, the formula is given in [6]. Although a few authors calculated the higher order corrections in some special cases [7, 8, 9], only the lowest order calculation is available in general to date.

Because of the polarization whose axis is anti-parallel to the beam axis, the polar angle ( $\theta$ ) becomes a natural quantization axis. We define the relative angle difference of the two particles as  $\theta_{e\gamma} = (\pi - \theta_e) - \theta_\gamma$  and  $\phi_{e\gamma} = (\pi + \phi_e) - \phi_\gamma$ , and we usually integrate out for the azimuthal angle ( $\phi$ ).

To compare the experimental data with the expectations, the detector efficiencies and resolutions have to be incorporated. The MEG detector and the trigger are optimized for the  $\mu^+ \rightarrow e^+\gamma$  events. The spectrometer preferentially selects high energy positrons, with  $E_e \gtrsim 45$  MeV. The photon energy is limited by the threshold set for the online trigger,  $E_\gamma \gtrsim 40$  MeV. The

trigger also requires a direction matching between the positron and the photon, which are emitted back-to-back in case of  $\mu^+ \rightarrow e^+\gamma$  event, resulting in deformation of the RMD distribution.

The Michel-positron spectrum is used as a calibration of the spectrometer. The resolution and energy-dependent efficiency are simultaneously extracted by fitting the function formed by folding the theoretical Michel spectrum with the detector response to the experimental spectrum. The absolute efficiency of positron measurement is not necessary because of the normalization scheme (described later).

The photon detector energy, timing and position resolutions as well as the energy scale are measured with photons from  $\pi^0$  decays in calibration sample. A pre-scaled trigger with a lowered  $E_\gamma$  threshold is enabled during the normal physics run. This allows a relative measurement of the energy-dependent efficiency curve of the photon detector, while the absolute efficiency is evaluated using the MC simulation and is cross-checked by the measurement of  $\pi^0$  decays.

The efficiency of the trigger direction matching is evaluated from the MC and the distribution of accidental background.

## 3. Measurement of RMD

The data sample analyzed in this paper corresponds to  $1.8 \times 10^{14}$   $\mu^+$  decays in the target, collected in 2009 and 2010. We used events reconstructed in the analysis window defined as  $45 < E_e < 53$ ,  $40 < E_\gamma < 53$  MeV,  $|\phi_{e\gamma}| < 0.3$  and  $|\theta_{e\gamma}| < 0.3$  rad. A complete description of MEG event reconstruction, event selection and analysis procedure is given in [2, 10].

The background of RMD analysis are from the accidental overlaps of positrons and photons originating from different muon decays. To measure the number of RMD events, we fitted a probability density functions (PDF), given by the sum of that of RMD (a sum of two Gaussians) and that of accidental background (a uniform distribution) to the  $t_{e\gamma}$  distribution (Fig. 1). To measure the distribution of RMD in energies and angles, the fits were repeated for data-sets divided into bins.

## 4. Results

It does not make sense to state the total branching ratio of RMD which is infrared divergent. Thus, one must always set a limit of the phase space to describe the branching ratio. Here, we measure the branching ratio for the largest phase space of our detector setup.

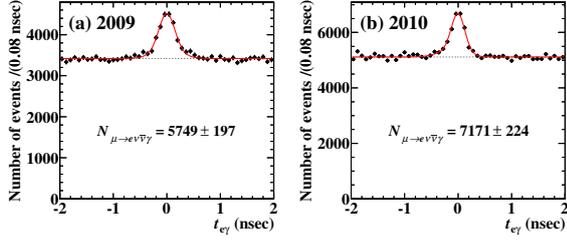


Figure 1: Distributions of  $t_{e\gamma}$  in (a) 2009 and (b) 2010. The best-fit functions of the sum of RMD and the accidental-background PDFs (red solid) and those of accidental-background only (dashed) are superimposed.

The RMD branching ratio is calculated by normalizing the observed number of RMD events to the number of Michel positrons counted simultaneously to be independent of the instantaneous beam rate and be nearly insensitive to the positron efficiency. The number of measured RMD events is  $12920 \pm 299$ . This corresponds to

$$\mathcal{B}(\mu \rightarrow e\nu\bar{\gamma}) = (6.03 \pm 0.14 \pm 0.53) \times 10^{-8} \quad \text{for } (E_e > 45, E_\gamma > 40 \text{ MeV}). \quad (1)$$

This is in good agreement with the theoretical calculation based on SM,  $\mathcal{B}^{\text{SM}}(\mu \rightarrow e\nu\bar{\gamma}) = 6.15 \times 10^{-8}$ . The largest contribution to the systematic uncertainty comes from the energy dependence of positron efficiency. Difficulties of its determination are there in the correlation between the acceptance curve and the response function as well as the dependence of the spectrum on the trigger direction-matching.

We also performed a standard  $\chi^2$ -fit to the measured spectrum with the polarization and the normalization as floating parameters in order to study the spectrum shape in the three-dimensional space  $(E_e, E_\gamma, \theta_{e\gamma})$ . The data sample was divided into  $2 \times 2 \times 6$  bins in  $(E_e, E_\gamma, \theta_{e\gamma})$  respectively (24 bins in total). Since the systematic uncertainties produce correlations among the bins, we built a covariance matrix calculated by the deviation of the expectation when parameters are varied. The  $\chi^2$  values for  $P = 0.9$  and for the best-fit value are  $\chi^2(P = 0.9)/\text{DOF} = 13.3/23$  and  $\chi^2(\text{min})/\text{DOF} = 11.9/22$ , respectively, where the normalization parameter is at the best-fit value for each case. These results show that the experimental spectrum shape is consistent with the SM predictions. The distribution of measured RMD events and the calculated ones are shown in Fig. 2.

The best-fit value of the polarization is  $P = 0.7 \pm 0.16$ . The best fit of normalization parameter, which is relative to the Michel normalization, is  $0.95 \pm 0.04$ . These results are consistent with values measured independently from

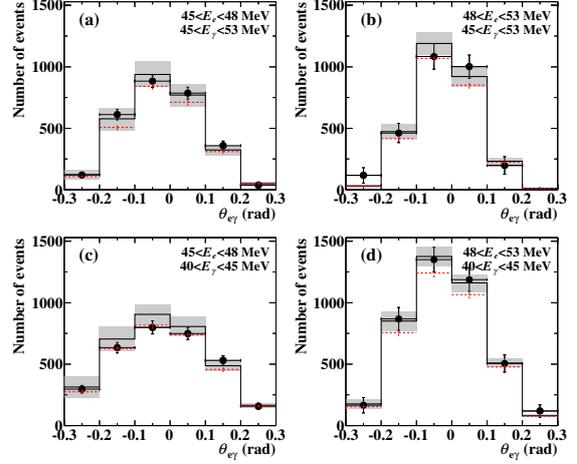


Figure 2: Distribution of RMD events in the 24 bins. The dots show the data and the histograms with solid line show expected distribution with  $P = 0.9$  normalized by Michel positron measurement. The red-dashed histograms show the best-fit distribution and the gray bands show the systematic uncertainty.

Michel positrons.

## 5. Discussion

The measurement of RMD is a powerful internal check of the experiment. The analysis in this paper uses the same data sample, calibrations, reconstruction and event selections as of the  $\mu^+ \rightarrow e^+\gamma$  search in [10]. Most part of the analysis was performed before unblinding the hidden box for the  $\mu^+ \rightarrow e^+\gamma$  search, in order to find possible mistakes in the procedure. Then, to get measurements of RMD branching ratio and distribution in agreement with the SM prediction, strongly demonstrate the validity of the search. A more practical purpose of analyzing RMD is to estimate the number of RMD events in the fit region of the  $\mu^+ \rightarrow e^+\gamma$  search. We extrapolate the number of RMD events measured in the low- $E_\gamma$  region (energy-sideband) to the fit region by using the ratio of the partial branching ratios and of the efficiencies. This estimate is directly integrated into the likelihood of the  $\mu^+ \rightarrow e^+\gamma$  search as a constraint on the number of RMD events. Another application of the RMD analysis is the use of RMD events as an alternative normalization channel. The advantage of using RMD is the better resemblance to the  $\mu^+ \rightarrow e^+\gamma$  decay compared to Michel decay, since not only a positron but also a photon from a muon decay are measured in the same data sample. The systematic uncertainties are independent of those of Michel-positron normalization scheme. An uncertainty of 10% was as-

signed to the normalization in the analysis in [3] by using the Michel channel only, while the uncertainty is now reduced down to 4% by the combination of the two channels.

Recently a precise measurement was reported by PI-BETA group [11]. This result has a lower uncertainty than ours. Nevertheless, our result is important because of the more stringent phase space. The energy and angular region where we are sensitive is close to the kinematical upper bound and is relevant to the background for  $\mu^+ \rightarrow e^+\gamma$  search. The largest data statistics and the higher timing resolution of MEG make it possible to study the details of this rare decay. In addition, this is the first measurement of RMD from polarized muon decay. We could test the Lorentz structure of the weak interactions through the investigation of new combination of coupling constants, which can not be accessed only from the measurement of non-radiative decay.

For better determinations of background and normalization for  $\mu^+ \rightarrow e^+\gamma$  search as well as a good test of SM, better theoretical calculation of  $O(1\%)$ , including one-loop corrections, applicable to the MEG experimental situation are desired.

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